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THERMODYNAMIC INVESTIGATION OF HEAT AND MASS TRANSFER IN A NONUNIFORM GAS SUSPENSION FLOW

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The effect of nonuniform motion on the transfer processes in a gas suspension flow is examined. It is shown that new inertial terms appear in the transfer equations together with the usual terms.

During nonuniform motion of a gas suspension flow, new forces which intensify the transfer processes appear, in addition to the thermodynamic forces recorded in [6]. In order to determine these forces, we examine a nonuniform, multicomponent gas flow containing suspended elementary nuclei, composed of solid-phase particles, surrounded by a gaseous envelope differing in composition from the carrier gas.

Let us extend the reduced-parameter method described in [1] to the disperse system under examination. Then the actual discrete system can be approximately described as a continuous, multicomponent gas mixture in which the individual components move at different velocities in relation to the center of mass of the system.

In accordance with [2], we take as the internal energy of unit mass of the multicomponent gas mixture the difference of total energy of the gas e, potential energy ψ, and the kinetic energy of all the components:

$$u^* = e - \sum_k \rho_{ko}\psi_k - \sum_k \frac{1}{2}\rho_{ko} w_k^2. \qquad (1)$$

Taking into consideration that

$$\sum_k \rho_{ko} w_k = w_c, \ \sum_k \rho_{ko} = 1,$$

we transform Eq. (1):

$$u^* = e - \psi - \frac{1}{2} w_c^2 - \sum_k \frac{1}{2}\rho_{ko}(\mathbf{w}_k - \mathbf{w}_c)^2. \qquad (2)$$

The first three terms of the right side of Eq. (2) represent the usual expression for the internal energy of the gas [6]; consequently, Eq. (2) may be written as

$$u^* = u - \sum_k \frac{1}{2}\rho_{ko}(\mathbf{w}_k - \mathbf{w}_c)^2. \qquad (3)$$

In order to determine the volume concentration of entropy production, we utilize Gibbs' equation for an open systen which, in the absence of chemical reactions, is written

$$T\frac{dS}{d\tau} = \frac{du^*}{d\tau} + p\frac{dv}{d\tau} - \sum_k \mu_k \frac{d\rho_{ko}}{d\tau}. \qquad (4)$$

Substituting into Eq. (4) the value of u* from (3) and taking into consideration that the diffusion mass flow of the k component is expressed relative to the motion of the center of mass of the system by the formula

$$\mathbf{j}_k = \rho_k(\mathbf{w}_k - \mathbf{w}_c),$$

and we obtain

$$T\frac{dS}{d\tau} = \frac{du}{d\tau} + p\frac{dv}{d\tau} - \sum_k \mu_k \frac{d\rho_{ko}}{d\tau} - \rho^{-1}\sum_k \mathbf{j}_k \frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}. \qquad (5)$$

Using formula (5), according to the method described in [6], we determine an expression for the total rate of entropy increase corresponding to both gaseous and solid phases per unit volume of the system (gas suspension). We assume that the gaseous envelope surrounding a particle of solid phase is gradually destroyed so that relative motion of the phases and interphase friction develops. In this case the entropy increase depends on both the work done by the forces of interphase friction and the acceleration of the gaseous envelope in the center of mass system

$$\frac{\partial}{\partial \tau}(\rho' dS + \rho_0' dS_0) = \frac{1}{T}\left\{\sum_i \sum_j (p\,\delta_{ij} - F_{ij})\frac{\partial w_j}{\partial x_i}\right\} -$$
$$- \mathbf{j}_q \frac{\operatorname{grad} T}{T^2} - \sum_k \mathbf{j}_k \left\{\operatorname{grad}\frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right\} - \tag{6}$$
$$- \rho_0' \mathbf{F}_f \frac{\mathbf{w}^*}{T} - \operatorname{div}\left(\frac{\mathbf{j}_q}{T} - \sum_k \mathbf{j}_k \frac{\mu_k}{T} + \rho' S w\right).$$

The term $\sum_k \mathbf{j}_k \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}$ on the right side of Eq. (6) characterizes the rate of entropy increase per unit volume of the system due to mass transfer connected with the variation in time of the velocity of the various components in the center of mass system.

From Eq. (6) we find the rate of entropy production per unit volume of gas suspension due to irreversible processes:

$$\Delta \dot{S} = \frac{1}{T}\left\{\sum_i \sum_j (p\,\delta_{ij} - F_{ij})\frac{\partial w_j}{\partial x_i}\right\} - \mathbf{j}_q \frac{\operatorname{grad} T}{T^2} -$$
$$- \sum_k \mathbf{j}_k \left\{\operatorname{grad}\frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right\} - \rho_0' \mathbf{F}_f \frac{\mathbf{w}^*}{T}. \tag{7}$$

From Eq. (7) the thermodynamic driving forces are found by the methods described in [2-6]:

$$\mathbf{X}_u = \mathbf{X}_q = -\frac{\nabla T}{T^2};\ \mathbf{X}_{mk} = -\left[\nabla \frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right];$$
$$\mathbf{X}_f = \frac{\mathbf{F}_f}{T};\ \mathbf{X}_{\eta\, ij} = \frac{1}{T}\frac{\partial w_j}{\partial x_i}. \tag{8}$$

As is evident from expression (8), the thermodynamic mass flow driving force $\mathbf{X}_{mk}$ consists of three terms, the first of which equals the chemical potential gradient, the second depends on the external force F_k, while the third, inertial term is determined by the acceleration of the component in the center of mass system.

All the thermodynamic forces, with the exception of the last, have the same tensor dimension and can be compared with each other.

In this connection, it is possible to write down for the disperse system in question the following system of linear transfer equations:

$$\mathbf{j}_q = -L_{uu}\frac{\nabla T}{T^2} - \sum_k L_{uk}\left\{\nabla\frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right\} - L_{uw}\mathbf{F}_f^*,$$
$$\mathbf{j}_i = -L_{iu}\frac{\nabla T}{T^2} - \sum_k L_{ik}\left\{\nabla\frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right\} - L_{iw}\mathbf{F}_f^*,$$
$$\rho\frac{\mathbf{w}^*}{T_0} = -L_{wu}\frac{\nabla T}{T^2} - L_{ww}\frac{\mathbf{F}_f^*}{T} - \sum_k L_{wk}\left\{\nabla\frac{\mu_k}{T} - \frac{\mathbf{F}_k}{T} + \right. \tag{9}$$
$$\left. + \frac{1}{T}\frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau}\right\}.$$

The transfer equations obtained differ from equations given in [6] in the presence of the inertial term $d(\mathbf{w}_k - \mathbf{w}_c)/d\tau$, which influences the interphase exchange processes in the gas suspension–heat transfer and diffusion.

In the presence of chemical reactions, as noted in [6], the right side of Eq. (7) contains an additional term corresponding to entropy production due to the chemical reactions, which equals $\frac{1}{T} \times \sum_j A_j \rho \frac{d\xi_j}{d\tau}$. In connection with this we get an additional thermodynamic driving force equal to A_j/T.

As pointed out in [6], this force cannot be compared with thermodynamic forces of vector type. However, the system of transfer equations (9) will still hold even in the presence of chemical reactions.

Consequently, in reacting gas-suspension systems in nonuniform motion the molecular heat and mass transfer processes should be examined in relation to each other on the basis of Onsager's equations, taking into account the effect exerted on them by the inertial transfer terms.

It is of interest to determine the effect on the intensity of transfer processes exerted by individual terms of the thermodynamic forces. For this purpose one can utilize the method of comparison, s, the essence of which will be explained with reference to the example of analysis of the heat transfer equation (first equation of system (9)).

The main term of this equation is the first term on the right side characterizing the conductive transfer.

We compare this term with the inertial term of thermodynamic diffusion force. In this connection we will examine only those cases when the superimposed flow, defined by the inertial term, is not less than 10% of the basic flow. This condition can be written

$$0.1 L_{uu} \frac{\nabla T}{T^2} \leqslant L_{uk} \frac{1}{T} \frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau} . \tag{10}$$

Transforming the expression (10), we obtain the condition for the inertial term to begin affecting the heat transfer process:

$$\frac{T}{\nabla T} a \geqslant 0.1 \frac{L_{uu}}{L_{uk}} , \tag{11}$$

where

$$a = \frac{d(\mathbf{w}_k - \mathbf{w}_c)}{d\tau} .$$

Similarly, we obtain a condition for taking into account interphase friction in the heat transfer calculations:

$$\frac{T}{\nabla T} \mathbf{F}_f^* \geqslant 0.1 \frac{L_{uu}}{L_{uw}} . \tag{12}$$

Correspondingly, in the mass transfer calculations the region of influence of the inertial term is determined from the expression

$$\frac{1}{\nabla \mu_k} a \geqslant 0.1, \tag{13}$$

and the region of influence of interphase friction from the expression

$$\frac{1}{\nabla \mu_k} \mathbf{F}_f^* \geqslant 0.1 \frac{L_{ik}}{L_{iw}} . \tag{14}$$

The quantities on the left-hand sides of expressions (11–14) are the values, characteristic for the systems under examination, of the reference temperature and its gradient, acceleration, friction force, and chemical affinity.

From the previously adopted model of gas-suspension flow it is evident that the inertial term and the term determined by interphase friction cannot simultaneously have an important effect on the transfer process, since in the first case, it is supposed that a stable gaseous envelope surrounds each solid particle, while, in the second, it is assumed that this envelope is absent and the solid particle surface is directly exposed to the gas.

The preferential areas of influence on the intensity of the transfer process of the inertial term and interphase friction may be estimated from expressions (11)–(14).

As an example, using formulas (11)–(14), we made calculations to find the conditions under which the inertial term begins to exert a substantial effect on the heat transfer process in the gas suspension.

The initial data for the calculation were: carrier gas—a two-component gas mixture, comprising air and water vapor; the water vapor is formed as a result of moisture evaporation from the surface of solid phase particles uniformly distributed in the gas volume; air temperature $T_a = 302°$ K; temperature of solid-phase particles $T_s = 300°$ K; gas pressure $p = 1.01325 \times 10^5$ N/m²; relative concentration of components in gas mixture $p_{1_0} = p_{2_0} = 0.5$.

According to [5], we take the following values for the kinetic coefficients: $L_{uu} = \lambda T^2$; $L_{uk} = D_T \rho \rho_{10} \rho_{20} T^2$. Values of the physical constants and coefficients are taken from [7] and [8].

After substitution in formula (11) and appropriate calculations, we obtain

$$\frac{a}{\nabla T} \geqslant 2.1 \text{ m}^2/\text{sec}^2 \cdot \text{degree}$$

or

$$a \geqslant 2.1 \nabla T \text{ m/sec}^2.$$

From the obtained expression, by assigning the value of ∇T, it is possible to determine the value of the characteristic acceleration of the system.

Notation

X—thermodynamic transfer force; μ—chemical potential; L—Onsager's kinetic coefficient; **j**—mass flux; $\mathbf{F}_i$ and $\mathbf{F}_{ij}$—forces of external and internal friction; S—volume concentration of entropy; x_i—Cartesian coordinates; δ_{ij}—Kronecker's delta; u—internal energy of body; e—total energy of body; ψ—potential energy of body; **w**—velocity; τ—time; ξ_j—parameter characterizing degree of completion of chemical reaction; ρ—density of medium; F_f—friction force between the surface of solid and liquid; T—temperature; $\rho_{k_0} = \rho_k/\rho$—relative concentration; p—pressure; A_j—chemical affinity of reaction j; v—specific volume. Indices: k—component of mixture; q—heat; u—internal energy; m—mass; j—designation of chemical reaction; *—relative value; c—center of mass of system.

REFERENCES

1. R. G. Mokadam, IFZh, no. 6, 1963.
2. S. R. deGroot, P. Mazur, Nonequilibrium Thermodynamics, [Russian translation], Izd-vo "Mir," 1964.
3. S. deGroot, Thermodynamics of Irreversible Processes," [Russian translation], Gostekhizdat, 1956.
4. A. V. Lykov and Yu. A. Mikhailov, Theory of Mass and Energy Transfer [in Russian], Izd. AN BSSR, 1959.
5. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat, 1963.
6. I. T. El'perin, IFZh, no. 12, 1963.
7. N. B. Vargaftik, Handbook of Thermophysical Properties of Gases and Liquids [in Russian], FM, 1963.
8. N. V. Tsederberg, Heat Conductivity of Gases and Liquids [in Russian], GEI, 1963.

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